Inventory Management

Chapter 10

Inventory Management
Two Questions

1. How much inventory should be ordered at one time?
2. When should an order be placed?

Management needs some decision rules

Stock Keeping Units (SKU's)

- Control of individual items
- EXAMPLE - Four different size bolts in two different thread configurations (fine and coarse) would represent eight different SKU's

Lot Size Decision rules

- Lot-for-lot (L4L)
- Fixed order quantity
- Periods of supply
- Dynamic order quantities

Lot for Lot (L4L)

- Order exactly what is needed
- Changes as requirements change
- Requires time-phased information
  - MRP or Master Schedule
- No unused inventory
- Most often used with 'A' items

Fixed Order Quantity

- Order the same amount each time
- Easy to understand
- Does not minimize costs
- Min-max system
  - Order when you go below the minimum
  - Order up to the max

Periods of Supply

- Order enough to cover a demand over a certain period of time
- Days of supply
- Period Order Quantity
  - discussed later

Economic Order Quantity

- Minimizes the total costs of Carrying and Ordering
- Assumptions
  1. Demand is relatively constant and known
  2. The item is purchased in lots or batches
  3. Preparation costs and ordering costs are constant and known
  4. Replacement occurs all at once

When not to use EOQ

- Make-to-order
  - When the customer sets the quantity needed
- Shelf life is short
- There are other limitations
  - Tool life is limited
  - Raw material batch size has limitations
    - E.g. Capacity of the kettle

Development of the EOQ Formula

Average inventory = $\frac{\text{order quantity}}{2}$

Number of orders / yr = $\frac{\text{annual demand}}{\text{order quantity}}$

Development of the EOQ Formula - Example

- Order Quantity (Q) = 200 units
- Usage = 100 units per week

Average inventory = $\frac{Q}{2} = \frac{200}{2}$ = 100 units

Number of orders per year = $\frac{\text{annual demand}}{\text{order quantity}}$

= $\frac{100 \times 52}{200}$

= 26 times per year

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Development of the EOQ Formula - Example

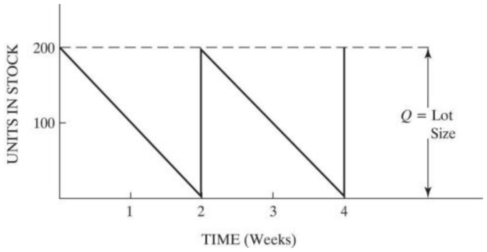

Figure 10.1 Inventory on hand over time

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Example Problem

Annual demand = 10,075 units per year
Order quantity = 650 units

Average inventory = Q/2 = 650/2 = 325 units

Number of orders per year
= A/Q = 10,075/650 =15.5 orders per year

Relevant Costs

- Annual cost of placing orders
- Annual cost of carrying inventory
- As the order quantity increases
  - the average inventory and the cost of carrying inventory increases
  - the number of orders per year and the annual cost of placing orders decreases

Relevant Costs

- Need to find the order quantity where:

the cost of carrying inventory
and the costs of ordering
will be a minimum

Inventory Costs

A = annual demand
S = ordering cost in dollars/order (setup)
i = annual carrying cost as a decimal or percentage (interest rate)
c = unit cost in dollars
Q = order quantity in units

Annual Ordering Cost

= (number of orders/year) x (cost/order)

$$= \frac{A}{Q} \times S$$

Annual Carrying Cost

= average inventory x cost of carrying one unit for one year

= average inventory x unit cost x carrying cost

$$= \frac{Q}{2} \times c \times i$$

Total annual Costs

The total of ordering and carrying costs

$$= \frac{A}{Q} \times S + \frac{Q}{2} \times c \times i$$

Example Problem

The annual demand is 10,000 units, the ordering cost $30 per order, the carrying cost 20%, and the unit cost $15. The order quantity is 600 units. Calculate:

a. Annual ordering cost
b. Annual carrying cost
c. Total annual cost

Example Problem - Solution

A = 10,000 units
S = $30
c = $15
i = 20% (.20)

It's a good idea to write these down every time

a. Annual ordering cost $= \frac{A}{Q} \times S$
= (10,000/600) x $30
= $500

Example Problem - Solution Continued

b. Annual carrying cost = $\frac{Q}{2}$ x c x i

= (600/2) x $15 x .20

= $900

c. Total annual cost

= cost of ordering + cost of carrying

= $500 + $900

= $1,400

Trial-and-Error Method

A = 1,000

S = $20 per order

c = $5 per unit

i = 20% (.20)

Cost of ordering = $\frac{1{,}000}{Q}$ x $20

Cost of carrying = $\frac{Q}{2}$ x $5 x .20

What happens to total cost when we change Q?

Trial-and-Error Method

Order Quantity (*Q*)	Ordering Costs (*AS/Q*)	Carrying Costs (*Qci*/2)	Total Cost
50	$400	$25	$425
100	200	50	250
150	133	75	208
200	100	100	200
250	80	125	205
300	67	150	217
350	57	175	232
400	50	200	250

Figure 10.2 Costs for different lot sizes

Total Costs and Order Quantity

- There is an order quantity which minimizes total costs
- The lowest cost occurs where the cost of ordering equals the cost of carrying
- The total cost varies little for a wide range of lot sizes about the minimum

Total Costs and Order Quantity

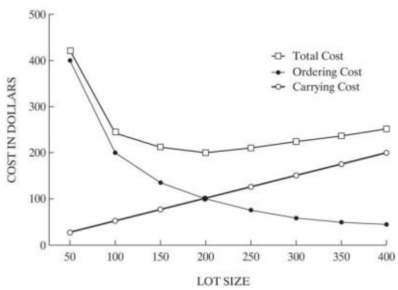


Figure 10.3 Cost versus lot size

Economic Order Quantity Formula

The EOQ is found by solving for the order quantity (Q) where the costs of carrying are equal to the costs of ordering

Economic Order Quantity Formula

$$\frac{Qic}{2} = \frac{AS}{Q}$$

$$Q^2 = \frac{2AS}{ic}$$

$$Q = \sqrt{\frac{2AS}{ic}}$$

$_{eo}Q$

the Quantity you order

EOQ
example from trial and error method

$$EOQ = \sqrt{\frac{2\,A\,S}{i\,c}}$$

$$= \sqrt{\frac{2x1{,}000x20}{.2 \times 5}}$$

$$= 200 \text{ units}$$

- A = 1,000 units
- S = $20
- i = .20
- c = $5

Costs
example from trial and error method

Carrying Cost

$$= \frac{Qic}{2}$$

$$= \frac{200 \times .2 \times \$5}{2}$$

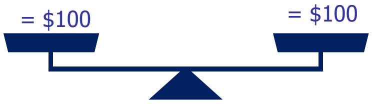
$$= \$100$$

Ordering Cost (annual)

$$= \frac{A\,S}{Q}$$

$$= \frac{1{,}000 \times \$20}{200}$$

$$= \$100$$



EOQ

- Minimizes Total Cost *of Carrying and Ordering*
- Cost to Order = Cost to Carry
- Used on 'C' items

Variations of the EOQ Model - Monetary Unit Lot Size

- Does not require unit cost (of each item)

A_D = annual usage in dollars
S = ordering cost in dollars *(no change)*
i = carrying cost as a decimal *(no change)*

$$EOQ = \sqrt{\frac{2AS}{i}}$$ (in dollars)

Quantity Discounts

- A discount is given for orders over a certain volume
- Must consider:
  - Purchase cost
  - Ordering cost (usually annual)
  - Carrying cost (also annual)
- If no order quantity is given, compare the discount quantity with the EOQ

Quantity Discounts

- Encourages buying more than normal
- Will increase the amount of inventory
  - Increases inventory carrying cost
  - Decreases ordering costs
- Savings from lower unit cost must be enough to offset higher carrying cost

Quantity Discount - Example

An item has an annual demand of 25,000 units, a unit cost of $10, an order preparation cost of $10, and a carrying cost of 20%. It is ordered on the basis of an EOQ, but the supplier has offered a discount of 2% on orders of $10,000 or more. Should the offer be accepted?

*Note the discount quantity is in dollars. It will be easier to calculate the EOQ in dollars.

Quantity Discount - Example continued

A_D = 25,000 x $10 = $250,000
S = $10
i = 20% = 0.2

$$EOQ = \sqrt{\frac{2 \times 250{,}000 \times 10}{0.2}} = \$5{,}000$$

Quantity Discount - Example continued

Discounted Order Quantity = $10,000 x 0.98 = $9,800

	No Discount	**With Discount**
Unit price	$10	$9.80
Lot Size	$5,000	$9,800
Avg Inventory Qc/2	$2,500	$4,900
# of Orders/yr A/Q	50	25
Purchase Cost	$250,000	$245,000
Carrying Cost (20%)	500	980
Ordering Cost ($10)	500	250
Total Cost	$251,000	$246,230

Quantity Discount - Example continued

- There is a savings in purchase cost
  $250,000 - 245,000 = $5,000
- Carrying cost increases
  $980 - 500 = $480
- Ordering cost decreases
  $500 - 250 = $250

Total cost decreases
  $251,000 - 246,230 = $4,770

Good Idea?

Use of EOQ When Costs Are Not Known

- Ordering costs and carrying costs are generally the same for a family of items
- Costs may not be easy to determine

$$Q = \sqrt{\frac{2 A_D S}{i}}$$

- S and *i* are the same for all items

Use of EOQ When Costs Are Not Known

Let $K = \sqrt{\frac{2S}{i}}$ *K is used as a constant*

Then $Q = K\sqrt{A_D}$

Also $Q = \frac{\text{annual demand}}{\text{orders per year}} = \frac{A_D}{N}$

Therefore $K\sqrt{A_D} = \frac{A_D}{N}$

Use of EOQ When Costs Are Not Known Example Problem

A family of products is each ordered 4 times per year which is not the EOQ. The cost of ordering and carrying are not known. Can a better decision rule be calculated?

Item	Annual Usage	Orders per yr	Present Lot Size	$\sqrt{A_D}$	$K = \frac{\sqrt{A_D}}{N}$
A	$10,000	4	$2,500	$100	25
B	400	4	100	20	5
C	144	4	36	12	3
		12	$2,636	$132	33

Average Inventory = Σ Q/2 = $2,636 / 2 = $1,318

Use of EOQ When Costs Are Not Known Example Problem

All items should have the same ordering and carrying costs and should have the same value for K. This doesn't seem to be true for these items. It would be better to use an average value for K.

$$K = \frac{\Sigma\sqrt{A_D}}{N} = \frac{132}{12} = 11$$

Use this value of K to calculate the individual order quantities

Use of EOQ When Costs Are Not Known Example Problem

Item	Annual Usage	Orders per yr	Present Lot Size	$\sqrt{A_D}$	New Lot Size = $K\sqrt{A_D}$	New Orders per year = $N = A_D/Q$
A	$10,000	4	$2,500	$100	$1100	9.09
B	400	4	100	20	220	1.82
C	144	4	36	12	132	1.09
		12	$2,636	$132	$1452	12.00

Average Inventory = $1,318 $726

The inventory has been reduced (45%) while the number of orders stays the same

Period Order Quantity (POQ)

- EOQ assumes demand is uniform
  - this is usually not true
- EOQ orders a quantity that attempts to balance the cost of ordering with the cost of carrying
- POQ sets a time interval that orders the same number of times per year as the EOQ
- POQ can reduce carrying costs especially with non-uniform demand

Period Order Quantity

$$POQ_{weeks} = \frac{EOQ}{\text{average weekly usage}}$$

Example

The EOQ for an item is 2800 units and the annual usage is 52,000 units. What is the POQ?

Average usage = 52,000 / 52 = 1,000 per week

POQ = 2800 / 1000 = 2.8 weeks ⟶ 3 weeks

When an order is placed the order quantity is set to cover requirements for the next 3 weeks

POQ Example Problem

MRP Record EOQ = 250 units

Week	1	2	3	4	5	6	7	8	9	10	Total
Net Requirements	100	50	150		75	200	55	80	150	30	890
Planned Order Receipts	250		250			250			250		
Ending Inventory	150	100	200	200	125	175	120	40	140	110	1360

POQ Method
Weekly demand = 890 / 10 = 89 units
POQ = 250 / 89 = 2.81 ⟶ 3 weeks

Week	1	2	3	4	5	6	7	8	9	10	Total
Net Requirements	100	50	150		75	200	55	80	150	30	
Planned Order Receipts	300				330			260			
Ending Inventory	200	150	0	0	255	55	0	180	30	0	870

Note, the total ending inventory is reduced from 1360 to 870 units over the 10 week period.

Practical Considerations When Using the EOQ

- Lumpy demand
- Anticipation inventory
- Minimum order quantities
- Transportation inventory
- Multiple order quantities

Practical Considerations When Using the EOQ

- Lumpy Demand
  - EOQ assumes continuous, constant demand
  - Better to use POQ
- Anticipation Inventory
  - planned buildup for future capacity or demand

Practical Considerations When Using the EOQ

- Minimum order quantities
  - suppliers may set a minimum order quantity
  - may be the value of the total order
  - often for 'C' items
- Transportation inventory
  - quantity ordered my be set by transportation considerations
  - similar approach to quantity discounts

Practical Considerations When Using the EOQ

- Multiple order quantities
  - constrained by package size
    - skid-load
    - carton / case
    - barrel
    - etc

Order Quantities - Summary

- EOQ assumes uniform demand
- Gives reasonable results
- The total cost curve is flat around the EOQ
  - good guesses are close to the optimum
- Costs of ordering and carrying can be difficult to determine
  - the EOQ concept can be used for groups of items
- Order quantity influences the costs of carrying and ordering
- Period Order Quantity is best for lumpy demand